Building Roller Coasters

How calculus is needed in roller coaster design

Part 1A: Introduction

Have you ever wondered what goes into designing a thrilling roller coaster? For example, have you ever experienced that rush of adrenaline as you plummet down a steep drop or twist through a high-speed turn? What makes certain roller coasters stand out from the rest—are they designed with specific elements to maximize excitement or is it all about the engineering? The activity below allows you to use concepts from calculus to design your own roller coaster.

Part 1B: Connecting Calculus and Coasters

This activity will focus on two core concepts discussed in most calculus classes: continuity and differentiability.

Continuity: continuous functions do not have any jumps or gaps.

Differentiability: a function is differentiable at a point when it is continuous and changes in a smooth manner without sudden changes in direction. In other words, the function must not have any sharp turns (like a "V" shape) at any point. The slope (i.e. the derivative) can be calculated at each point.

On a roller coaster track, continuity and differentiability would ensure that the path is smooth, without sharp angles or breaks. The coaster would move in a smooth manner that helps riders maintain a sense of speed and excitement while avoiding jarring movements or even crashes.

Part 1C: Designing a Coaster

When designing your first roller coaster, you will create up to three functions, that when combined, will represent a roller coaster path that is continuous and differentiable. This means the track (the combination of all three functions) must flow smoothly without any breaks or sharp turns. You'll need to submit the following:

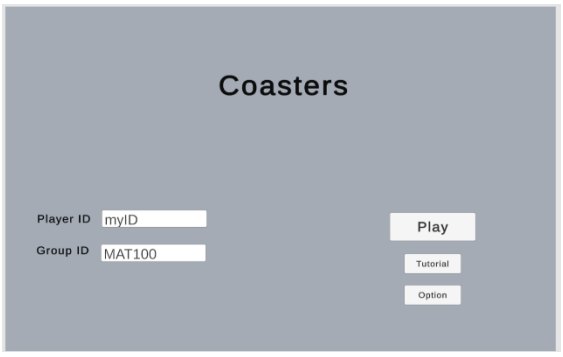
1. The functions that represent the track.
2. Calculations showing how the track is continuous and differentiable at the points where the three functions connect.
3. A screenshot of your coaster design and your final score.

You can build and visualize your coaster here: Roller Coaster 9 by minh-nguyen-mn

Password: kuiperresearch

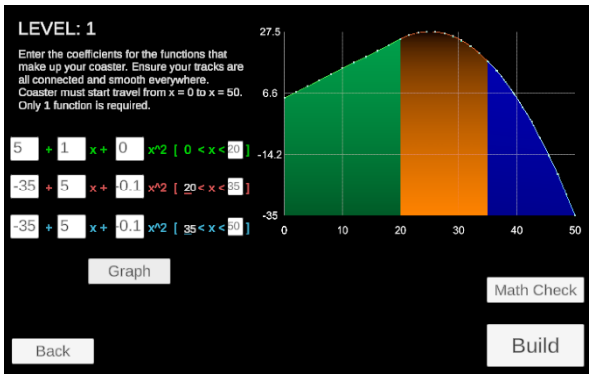
Here are some key terms that will be recorded during the coaster game:

- The game will ask you for a **Player ID**. You can use any name you like, but this will be on the internet. Use a code name that you will remember, but do not use a name that will easily identify you.
- Use the **Group ID** : **Testing**
- Click the **Play** button, **Level 1** and then enter your roller coaster functions.



Your **goal** is to enter a smooth and continuous function that represent the most exciting roller coaster possible. The final score represents the level of excitement which is measured by the total changes in elevation of your coaster.

Part 1D: Example:



In the image to the right, we have created a simple roller coaster.

1. The first function (the green line) shows the lift ramp of a roller coaster. Here we entered a simple linear function (y = 5 +1x). At the start of the roller coaster (x = 0), people would load at a height of y = 5. Typically, a coaster will have some type of conveyor to get you to the top of the coaster, then gravity will carry you through the rest of the coaster.
2. The equation entries require that the ending x value of the first equation is the starting value of the next equation. This ensures that there are **no horizontal gaps** in the coaster.
3. The **Graph** and **Math Check** buttons allow us to verify that the coaster will work. To be continuous the y value of the first function when x = 20 must equal the y value of the second function when x = 20. This ensures that there are **no vertical gaps** in the coaster.
4. The second function is quadratic, allowing the coaster to have a smooth curve. If two linear equations are put next to each other, the graph would have a sharp V-shaped curve, meaning that it would not be differentiable (and not safe to ride).
5. In order to be differentiable the slope of the first line and the slope of the second line must also be equal at x = 20.
6. Here the second and third functions are the same (except for the x-axis), this is not necessary, just two equations could be listed and the cells in the last row can all be blank. To get more points, more curves are needed.

In this version of the game, there are some restrictions:

1. You can enter 1, 2, or 3 functions.
2. The x-axis must go from 0 to 50
3. The height of the coaster must always be greater than 0
4. Before building your coaster, you should use the Math Check button to ensure that your functions are continuous and differentiable.
5. Your score, which is based on elevation changes, only starts after your coaster starts changing direction. In other words, if you just draw a straight line that only goes up, your score will be 0.

Part 1D: Comparing Coasters

How fun do you think your coaster would be? Before we come to any conclusions, use the following app to look at the other coasters people in your class made. https://shiny.grinnell.edu/coaster/

- In the **Group ID** section, make sure only the Group ID for your class is selected.
- Check the "show high score table" option.
- Which Player ID had the highest score in your class?
    - What strategies appear to be the most important in getting a high score?
    - Does their coaster look practical to build in real life?

- What restrictions would make the coaster game more practical?

Part 1E: Discussion Questions

- If you were to design a new roller coaster game, what restrictions would you have?
  - Does it make sense to assume the function can't go below 0?
  - Should we require that when x = 0 the height should be 0 as well?
  - Should we require that the beginning and ending of the track needs to be horizontal?
  - Should the first hill be higher than all the others? Why is this typically important?
  - Discuss any other rules you think should be implemented.
- Would you expect to have a more exciting coaster if you used cubic and exponential equations instead of linear and quadratic? Explain why.